

Large Skyrmions

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- ▶ Skyrme field is SU(2)-valued:

$$U(\mathbf{x}) = \sigma(\mathbf{x}) \mathbf{1} + i\pi(\mathbf{x}) \cdot \boldsymbol{\tau}$$

where $\sigma(\mathbf{x})^2 + \pi(\mathbf{x}) \cdot \pi(\mathbf{x}) = 1$.

- ▶ Baryon number:

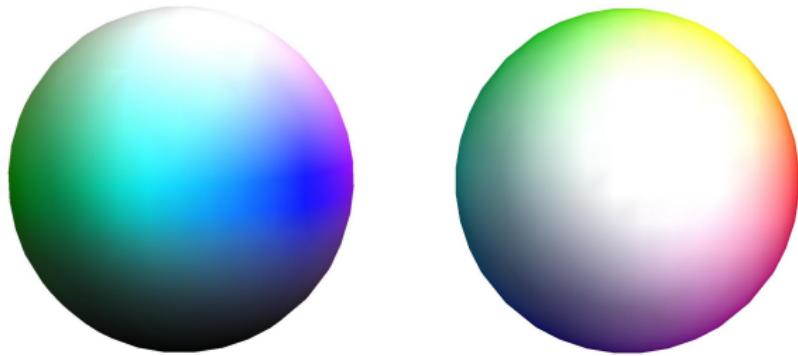
$$B = -\frac{1}{24\pi^2} \int_{\mathbb{R}^3} \varepsilon_{ijk} \text{Tr}(R_i R_j R_k) d^3x$$

where $R_i = (\partial_i U) U^{-1}$.

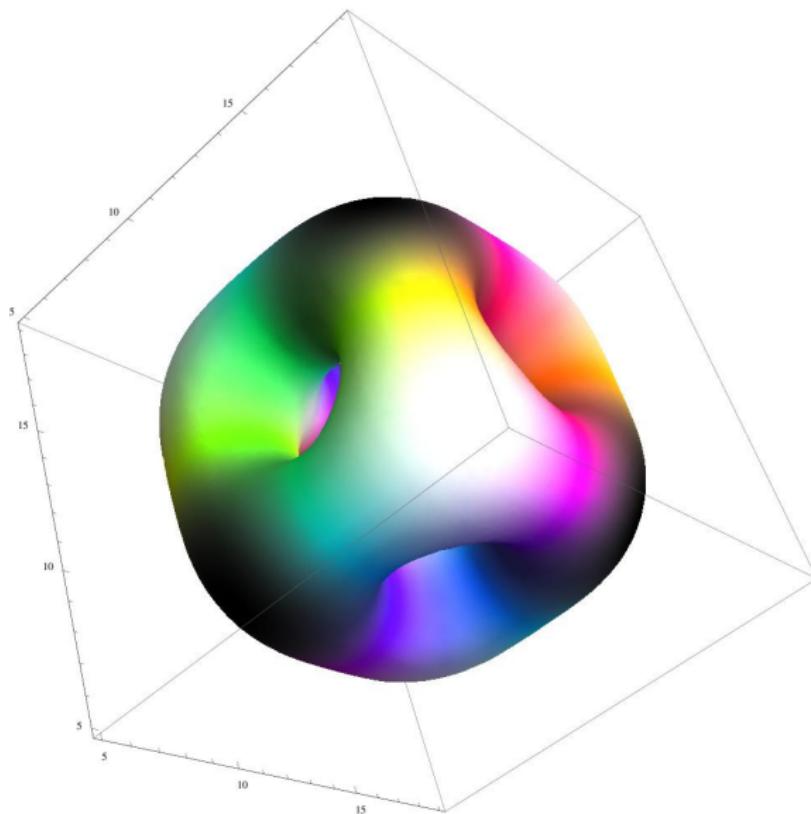
- ▶ Skyrmions minimise the (static) Skyrme energy:

$$E = \int_{\mathbb{R}^3} \left\{ -\frac{1}{2} \text{Tr}(R_i R_i) - \frac{1}{16} \text{Tr}([R_i, R_j][R_i, R_j]) + m^2 \text{Tr}(\mathbf{1} - U) \right\} d^3x$$

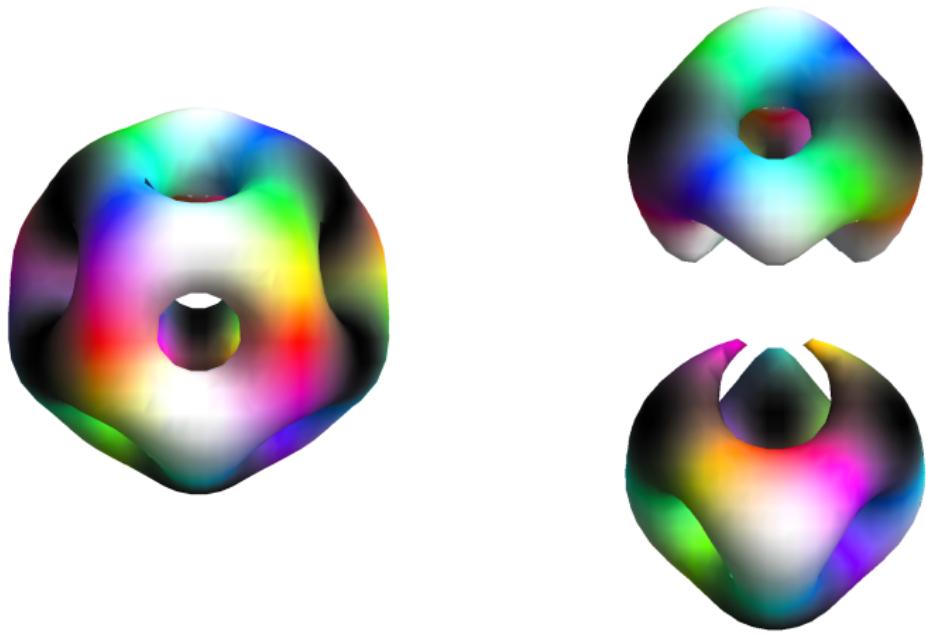
- ▶ Visualising Skyrmions – the Runge colour sphere records the unit pion field $\pi/|\pi|$. The colours are superposed on a constant energy density surface.



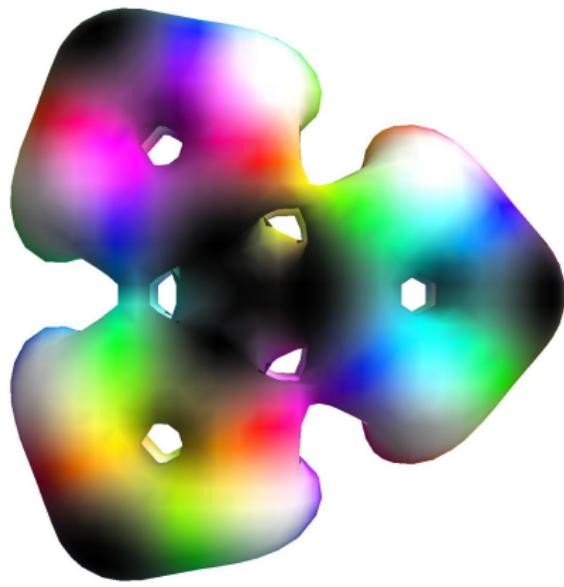
$B = 1$ Skyrmion (two different orientations)



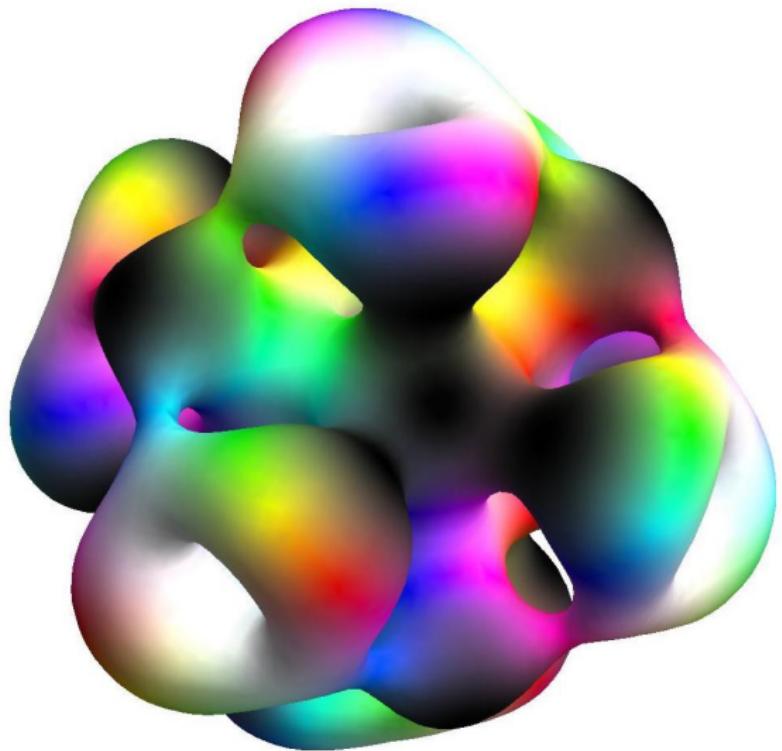
$B = 4$ Skyrmion – Alpha particle



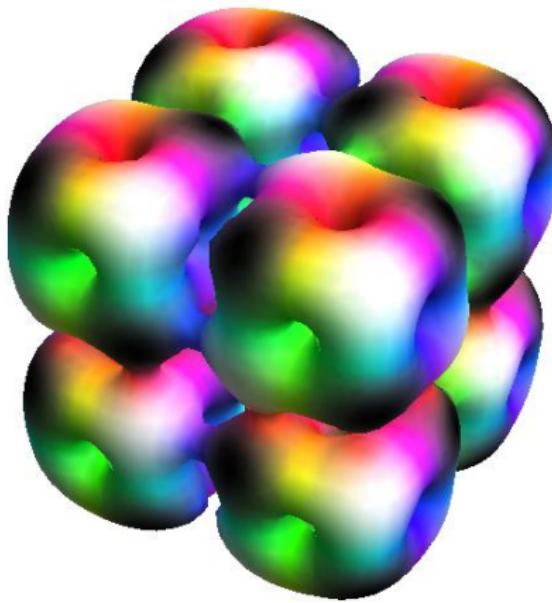
$B = 7$ Skyrmion and its deformation into clusters



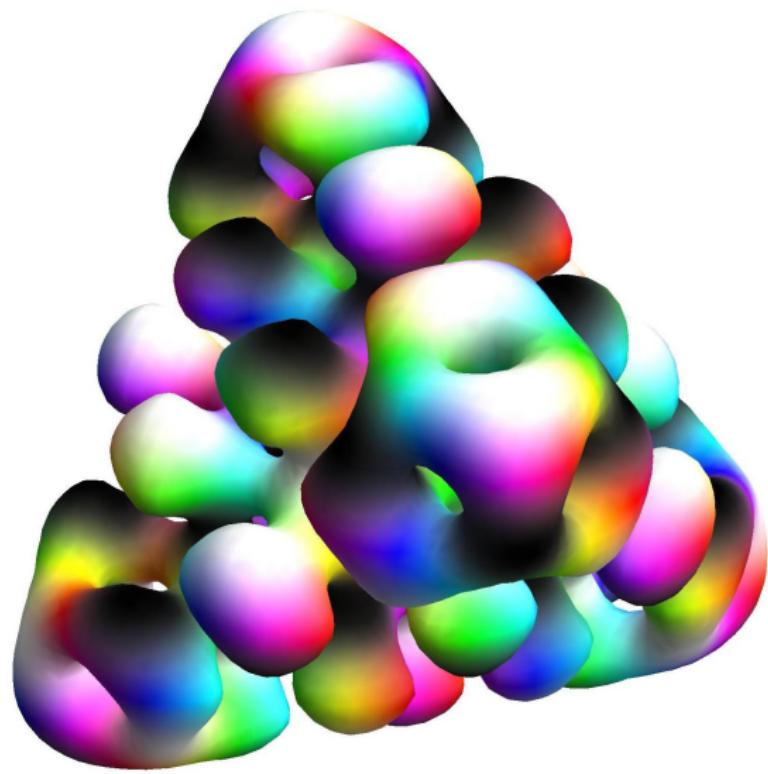
$B = 12$ Skyrmion



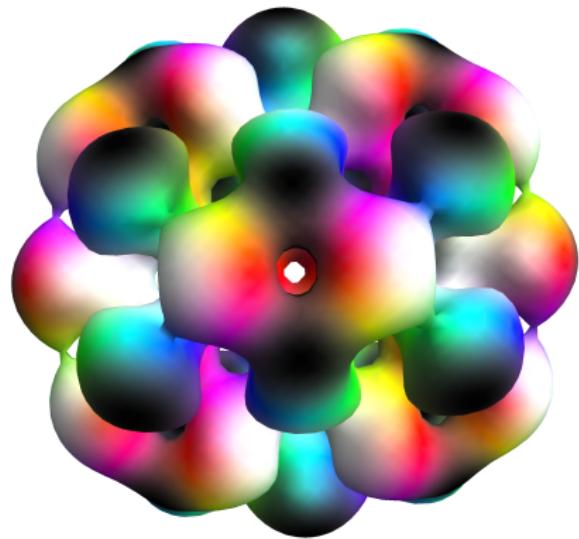
$B = 16$ Skyrmion



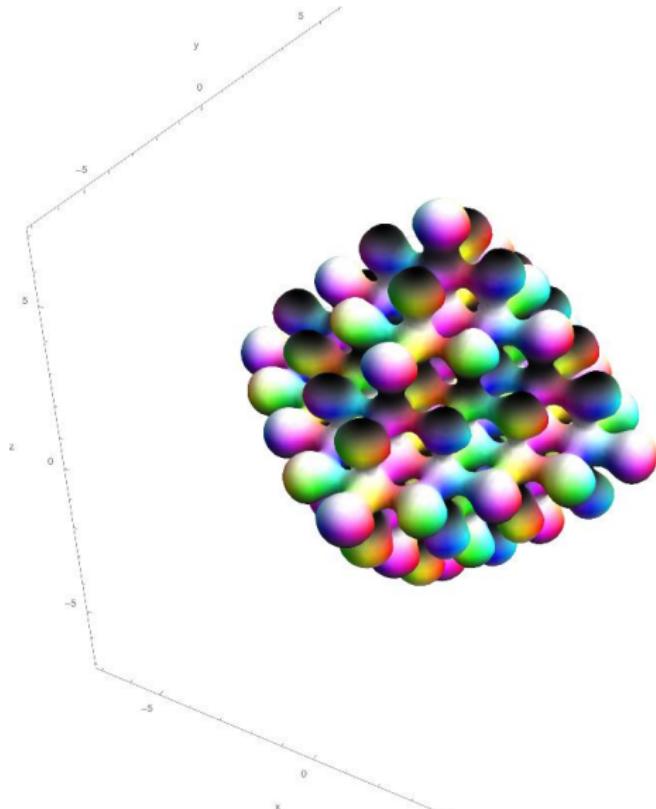
$B = 32$ Skyrmion



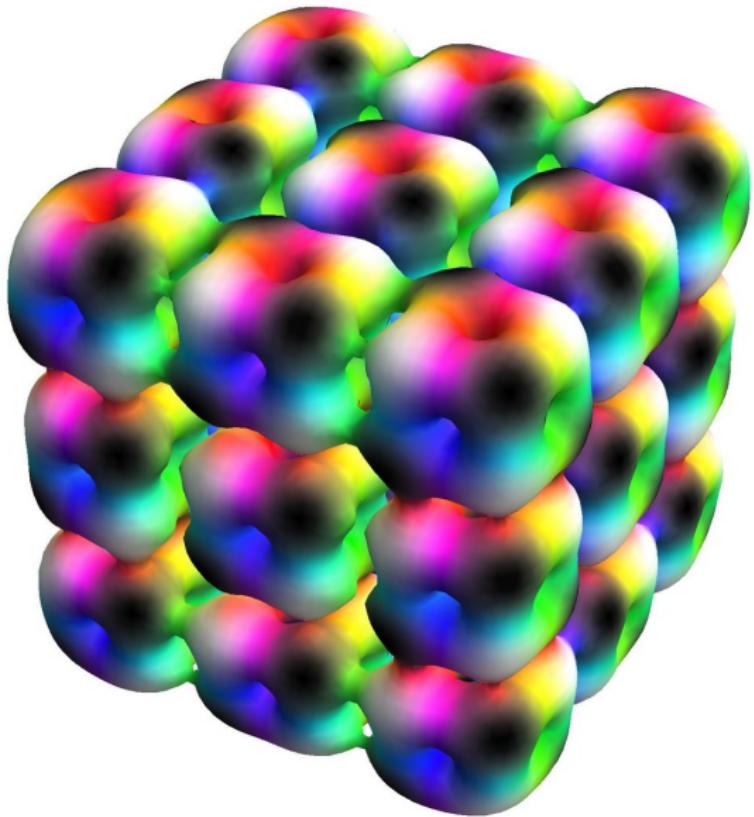
$B = 35$ Skyrmion



$B = 40$ Skyrmion



$B = 85$ Skyrmion



$B = 108$ Skyrmion

Quantized Skyrmions

- ▶ Skyrmiон quantum states model protons, neutrons and larger nuclei.
- ▶ Skyrmiонs can be quantized as (coloured) rigid bodies; they acquire spin and isospin. Skyrme field topology and Skyrmiон symmetries constrain the allowed states [Finkelstein-Rubinstein].
- ▶ Quantized vibrational deformations of Skyrmiонs have also been considered.

Bethe–Weizsäcker Mass Formula

- ▶ Z = Proton number, N = Neutron number, I = Isospin.
- ▶ $Z + N = B$, $Z - N = 2I_3$
- ▶ A nucleus in its ground state has $I = |I_3|$.
- ▶ Nuclear mass = $Z M_p + N M_n - E_{\text{binding}}$

- ▶ Binding energy:

$$E_{\text{binding}} = a_V B - a_S B^{2/3} - a_C \frac{Z(Z-1)}{B^{1/3}} - a_A \frac{(2I)^2}{B} + \delta$$

- ▶ The terms describe effects of (V) Volume, (S) Surface, (C) Coulomb energy, (A) Proton-Neutron asymmetry. δ mops up various corrections.
- ▶ Can we estimate the coefficients a_V, a_S, a_C, a_A using Skyrmions?
- ▶ Electric charge density of Skyrmion is half the baryon density plus a contribution from I_3 isospin density.